

Generalized Chaplygin gas with $\alpha = 0$ and the Λ CDM cosmological model

J. C. Fabris*, S.V.B. Gonçalves† and R. de Sá Ribeiro‡

Universidade Federal do Espírito Santo, Departamento de Física
Av. Fernando Ferrari s/n - Campus de Goiabeiras, CEP 29060-900, Vitória, Espírito Santo, Brazil

February 2, 2008

Abstract

The generalized Chaplygin gas model is characterized by the equation of state $p = -\frac{A}{\rho^\alpha}$. It is generally stated that the case $\alpha = 0$ is equivalent to a model with cosmological constant and dust (Λ CDM). In this work we show that, if this is true for the background equations, this is not true for the perturbation equations. Hence, the mass spectrum predicted for both models may differ.

PACS number(s): 98.80.Bp, 98.65.Dx

The generalized Chaplygin gas (*GCG*) [1] is a recent proposal in order to explain the observed acceleration of the Universe [2, 3]. It is an exotic fluid with negative pressure whose equation of state is given by

$$p = -\frac{A}{\rho^\alpha} \quad , \quad (1)$$

with $0 \leq \alpha \leq 1$. This exotic fluid has been considered as an alternative to quintessence [4] and to the cosmological constant [5], which are other serious candidates to explain the accelerated expansion of the Universe. Many observational constraints have been obtained for cosmological models based on the *GCG*. One interesting aspect of such exotic fluid is connected with a possible unification of dark matter and dark energy through a simple fluid described by the equation of state (1).

Some authors (see, for example, reference [6]) claim that the comparison with observation indicates that α is peaked around zero. At same time, it is generally argued in the literature that the case $\alpha = 0$ is equivalent to a Λ CDM model [7, 8, 6]. The aim of the present letter is to show that, if this is true for the background solutions, it is not true for the linearized equations. In this sense, in what concerns the type Ia supernovae data both models leads to the same results, as well as for the position of the acoustic peaks in spectrum of the anisotropy of the cosmic microwave background radiation. But we expect a disagreement concerning the predictions for the mass spectrum.

The case $\alpha = 0$ means that the pressure remains constant as the Universe expands and the density decreases. Since, for this case, $p = -A$, the equations of motion for an isotropic and homogeneous Universe described by the flat Friedmann-Robertson-Walker metric are:

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho \quad , \quad (2)$$

*e-mail: fabris@cce.ufes.br

†e-mail: sergio@cce.ufes.br

‡e-mail: rosilene-ribeiro@bol.com.br

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = 8\pi G A \quad , \quad (3)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho - A) = 0 \quad . \quad (4)$$

These equations admit the solution,

$$a(t) = a_0 \sinh^{2/3}(Dt) \quad , \quad (5)$$

$$\rho(t) = A \coth^2(Dt) \quad , \quad (6)$$

where $D = \sqrt{6\pi G A}$. The scale factor has exactly the same behaviour as in a model with a dust fluid and a cosmological constant. In fact, the integration of equation (4) leads to the expression

$$\rho(t) = A + \rho_0 a^{-3} \quad . \quad (7)$$

Substituting this expression in equation (2), we obtain the same relation for the scale factor of the Λ CDM model. Notice that the solution (5) display a dust behaviour when $t \rightarrow 0$, and a cosmological constant behaviour when $t \rightarrow \infty$.

Since the equation of motion for the scale factor is the same as in the Λ CDM, the luminosity-distance and angular diameter relations will be same as in the Λ CDM model. Hence, the type Ia supernovae data and the location of the acoustic peaks in the CMB anisotropy spectrum are the same in both cases. But, the linearized equations around the background described above are quite different, as it will be verified now.

Introducing fluctuations around the background specified above, and using the synchronous coordinate condition $h_{\mu 0} = 0$, we end up with the following coupled system of equations for the GCG with $\alpha = 0$:

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} = 8\pi G \delta\rho \quad , \quad (8)$$

$$\dot{\delta} + 3\frac{\dot{a}}{a}\frac{A}{\rho}\delta = \frac{1}{2}\left(1 - \frac{A}{\rho}\right)\dot{h} \quad , \quad (9)$$

where

$$h = \frac{h_{kk}}{a^2} \quad \text{and} \quad \delta = \frac{\delta\rho}{\rho} \quad . \quad (10)$$

Using the background solutions and combining both equations, we find the following equation for density contrast δ :

$$\ddot{\delta} + \frac{4}{3}\left(\coth t + 3\tanh t\right)\dot{\delta} + 2\left(\tanh^2 t - \frac{1}{3}\coth^2 t + \frac{8}{3}\right)\delta = 0 \quad . \quad (11)$$

In this equation, the time coordinate has been re-scaled: $Dt \rightarrow t$. This equation for the density contrast can be compared with that for the Λ CDM model:

$$\ddot{\delta} + \frac{4}{3}\coth t \dot{\delta} - \frac{2}{3}\coth^2 t \delta = 0 \quad . \quad (12)$$

Equations (11,12) exhibit the same solutions in the “dust phase”, when $t \rightarrow 0$: $\delta_+ \propto t^{2/3}$ and $\delta_- \propto t^{-1}$. But, in the “cosmological constant phase” the behaviour of (11,12) are quite different. In fact, in this limit, those equations simplify to

$$\ddot{\delta} + \frac{16}{3}\dot{\delta} + \frac{20}{3}\delta = 0 \quad (GCG) \quad , \quad (13)$$

$$\ddot{\delta} + \frac{4}{3}\dot{\delta} - \frac{2}{3}\delta = 0 \quad (\Lambda\text{CDM}) \quad . \quad (14)$$

The equations admit solutions under the form of $\delta \propto e^{nt}$. For the generalized Chaplygin gas model with $\alpha = 0$, $n_+ = -5/3$ and $n_- = -2$, while for the Λ CDM model, $n_{\pm} = -2/3 \pm \sqrt{10}/3$. Hence, in the Λ CDM phase there is a growing mode even during the “cosmological constant phase”. On the other

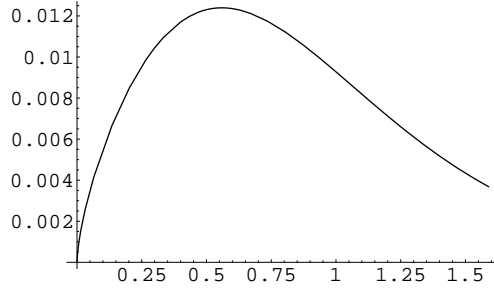


Figure 1: Behaviour of the density contrast for the *GCG* model with $\alpha = 0$.

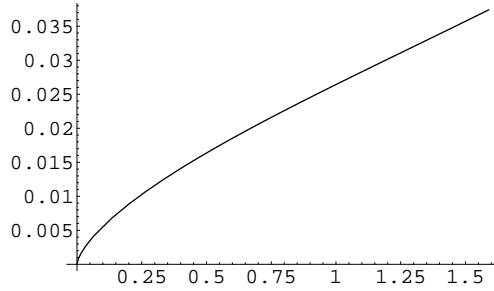


Figure 2: Behaviour of the density contrast for the Λ CDM model.

hand, for the generalized Chaplygin gas model, there are only decreasing modes in the “cosmological constant phase”. This agrees with the perturbative analysis made for the Chaplygin gas with $\alpha = 1$ [9, 10].

Equation (11) seems to admit no closed solutions. Hence, a numerical integration must be performed. Frequently, in order to fix the interval of integration, the functions are re-expressed in terms of the redshift $z = -1 + a/a_0$, where a_0 is the scale factor today. But, here, we will keep working with the re-scaled cosmic time. The age of the Universe today will be fixed through the deceleration parameter

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \quad , \quad (15)$$

and the integration will be performed from the beginning of the matter dominated phase, which occurs about $z \sim 4,000$. Estimations of the deceleration parameter are consistent with a value $q = -0.77$ [11]. This leads to $t_0 \sim 1$; the initial time corresponding to $z = 4,000$ is $t_i \sim 10^{-5}$. At this initial moment, the density contrast will be assumed to have a scale invariant spectrum with $\delta_i = 10^{-5}$.

The result of the numerical integration for the *GCG* model with $\alpha = 0$ is displayed in figure 1, while the result for the Λ CDM model is shown in figure 2. The main difference is due to the fact that, while for Λ CDM the density contrast for the dust fluid keeps growing even during the “cosmological constant phase”, for the *GCG* with $\alpha = 0$ it stops growing and begins to decrease. The numerical integration confirms the asymptotic analysis performed above.

It is curious to remark that the same behaviour for the background may be found when the matter content of Universe is described by a viscous fluid, if the bulk viscosity varies inversely with density and if the spatial section is flat. In such a situation, the background equations read

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho \quad , \quad (16)$$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p^*) = 0 \quad , \quad (17)$$

where $p^* = p - 3\frac{\dot{a}}{a}\xi(\rho)$. Fixing $p = 0$, and $\xi(\rho) = \xi_0\rho^\nu$, the equations above become equivalent to the *GCG* model if $\nu + 1/2 = -\alpha$. The case $\alpha = 0$ is reproduced if $\nu = -1/2$. Again this equivalence is valid only for the background relations: at perturbative level, this exotic viscous model is inequivalent both to the corresponding *GCG* model and to Λ CDM.

The main consequence of this quite different behaviour concerns the power spectrum for the mass distribution of the Universe. Observational results on the mass power spectrum have been obtained through the 2dFGRS program [12], up to scale of some hundreds of megaparsecs. A proper comparison with these observational data implies, among other things, to evaluate the transfer function for the *GCG* with $\alpha = 0$. The evaluation of the transfer function requires to consider a more detailed model, including for example the radiative component of the matter content of the Universe [13]. We hope to present a full analysis of this problem in the future.

Acknowledgements: We thank CNPq (Brazil) for partial financial support to this work.

References

- [1] M.C. Bento, O. Bertolami and A.A. Sen, Phys. Rev. **D66**, 043507 (2002);
- [2] A.G. Riess et al, Astron.J. **116**, 1009(1998);
- [3] S. Perlmutter et al, Astrophys. J. **517**, 565(1998);
- [4] R.R. Caldwell, R. Dave and P. Steinhardt, Phys. Rev. Lett. **80**, 1582(1998);
- [5] V. Sahni, *The cosmological constant problem and quintessence*, astro-ph/0202076;
- [6] M. Makler, S.Q. de Oliveira and I. Waga, *Observational constraints on Chaplygin quartessence: background results*, astro-ph/0306507;
- [7] P.P. Avelino, L.M.G. Beça, J.P.M. de Carvalho, C.J.A.P. Martins and E.J. Copeland, *The onset of the non-linear regime in unified dark matter models*, astro-ph/0306493;
- [8] J.V. Cunha, J.S. Alcaniz and J.A.S. Lima, *Cosmological constraints on Chaplygin gas dark energy from galaxy clusters X-ray and supernova data*, astro-ph/0306319;
- [9] A. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. **B511**, 265(2001);
- [10] J.C. Fabris, S.V.B. Gonçalves and P.E. de Souza, Gen. Rel. Grav. **34**, 53(2002);
- [11] R. Colistete Jr, J. C. Fabris, S.V.B. Gonçalves and P.E. de Souza, *Bayesian Analysis of the Chaplygin Gas and Cosmological Constant Models using the SNe Ia Data*, astro-ph/0303338;
- [12] W.J. Percival et al., *Parameter constraints for flat cosmologies from CMB and 2dFGRS power spectra*, astro-ph/0206256;
- [13] P. Coles and F. Lucchin, **Cosmology**, Wiley, New York(1995).